

Topological structure in the $SU(2)$ vacuum^{*†}

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We study the topological content of the vacuum of $SU(2)$ pure gauge theory using lattice simulations. We use a smoothing process based on the renormalization group equation. This removes short distance fluctuations but preserves long distance structure. The action of the smoothed configurations is dominated by instantons, but they still show an area law for Wilson loops with an unchanged string tension. The average radius of an instanton is about 0.2 fm, at a density of about 2 fm^{-4} .

Based on phenomenological models, it has been argued that instantons are largely responsible for the low energy hadron and glueball spectrum [1]. Instanton liquid models attempt to reproduce the topological content of the QCD vacuum and conclude that hadronic correlators in the instanton liquid show all the important properties of the corresponding full QCD correlators. These models appear to capture the essence of the QCD vacuum, but their derivations involve a number of uncontrolled approximations and phenomenological parameters.

Lattice methods are the only ones we presently have, which might address this connection starting from first principles. Lattice studies of instantons can suffer from several difficulties. An unambiguous topological charge can be assigned only to continuous gauge field configurations living on a continuum space-time. On the lattice, the charge can only be defined as that of an interpolated continuum field configuration. This interpolation however is non-unique on Monte Carlo generated lattice configurations.

Another problem is connected to the fact that while the continuum gauge field action is scale invariant, the lattice regularisation breaks this invariance and the action of lattice instantons typically depends on their size. This might distort the size distribution of instantons and in particular can lead to an overproduction of small in-

stantons which can even spoil the scaling of the topological susceptibility.

The framework of classically perfect fixed point (FP) actions [2] is particularly suitable to address these problems. Fixed point actions can be shown to have scale invariant instanton solutions and there are no charge 1 objects with an action lower than the continuum instanton action. The FP context also gives a consistent way of interpolation to define the topological charge.

The fixed point action for any given configuration V on a lattice with lattice spacing a is defined by the weak coupling saddle-point equation

$$S^{FP}(V) = \min_{\{U\}} (S^{FP}(U) + T(U, V)), \quad (1)$$

where T is the blocking kernel of a real-space RG transformation from the fine lattice (spacing $a/2$) to the coarse lattice (a) and the minimum is taken over all fine lattice configurations U . The minimising fine configuration U_{\min} is the smoothest possible of those U 's that block into V . It is a very special interpolating configuration which, in the weak coupling limit, gives the largest contribution to the path integral defining the RG transformation for the given coarse configuration V . Finding U_{\min} for a given V will be referred to as "inverse blocking". In principle inverse blocking can be repeated several times until the resulting configuration on the finest grid becomes smooth enough that any "sensible" definition of the topological charge gives the same integer value.

In the remainder of the paper we discuss the implementation of the above ideas for the 4d

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SU(2) gauge theory. For more details we refer the reader to [3]. Another implementation can be found in Ref. [4]. Our calculations were performed at several lattice spacings between 0.1–0.18 fm with an approximate FP action. At these values of the lattice spacing the total charge is well defined already after one step of inverse blocking but individual instantons cannot be identified at this stage. Further iteration of the inverse blocking is presently impossible due to computer memory limitations. Therefore we used a smoothing cycle based on inverse blocking followed by a blocking step but on a different coarse sublattice, diagonally shifted by $a/2$.

In order to understand how this works we note that, although the inverse blocked lattice has a physical lattice spacing $a/2$ as measured by any long-distance observable, locally it is much smoother than typical Monte Carlo generated lattice configurations with the same lattice spacing. Nevertheless this locally very smooth configuration, by construction, still blocks back into the given V . This is ensured by the delicate coherence present in the U_{\min} configuration on a distance scale of a . The diagonal shift by $a/2$ before blocking destroys exactly this coherence and as a result the shifted U_{\min} blocks into a configuration that is locally much smoother than V was.

After one such cycling step the shortest distance fluctuations are considerably reduced but long-distance features are preserved. Since the lattice size does not change, this step can be repeated several times. The stability of the long distance physical properties of the configurations can be demonstrated by the invariance of the string tension (see Fig.1). The total topological charge — as measured in each step on the fine grid — as well as artificially laid down instantons and I-A pairs also turned out to be unchanged by cycling. After about 6 cycles the locations and sizes of individual instantons could be identified.

While their locations were quite stable from the stage where we could reliably identify them, the size of some instantons kept changing slowly throughout the iteration. This was taken into account by extrapolation when measuring sizes. This might explain why on one of our configurations Ref. [5] found that the topological charge

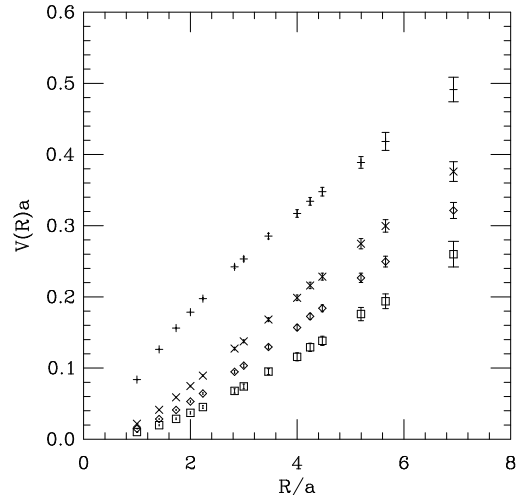


Figure 1. Potential measured on the inverse blocked 16^4 $\beta = 1.5$ configurations after performing different numbers of smoothing cycles (plusses 1 cycle, crosses 3 cycles, diamonds 5 cycles and squares 9 cycles).

— as measured with the fermionic overlap — changed over cycling. It is very likely that over repeated cycling an already existing small instanton grew above the threshold below which the overlap cannot detect small instantons.

The instanton size distribution obtained at different values of the lattice spacing — as defined by the Sommer parameter — is shown in Fig. 2. We obtain a topological susceptibility

$$\chi_t = \frac{\langle Q^2 \rangle}{V} = (230(10) \text{ MeV})^4, \quad (2)$$

which is about 20% larger than the values obtained with improved cooling [6] and the heating method [7].

The smoothed configurations have essentially the same long-distance physical properties as the unsmoothed ones. On the other hand about 70% of their action can be accounted for by the instantons. This alone suggests that instantons might explain most of the long-distance features of QCD. The most straightforward way to check this is to prepare artificial configurations by laying down instantons in exactly the same way

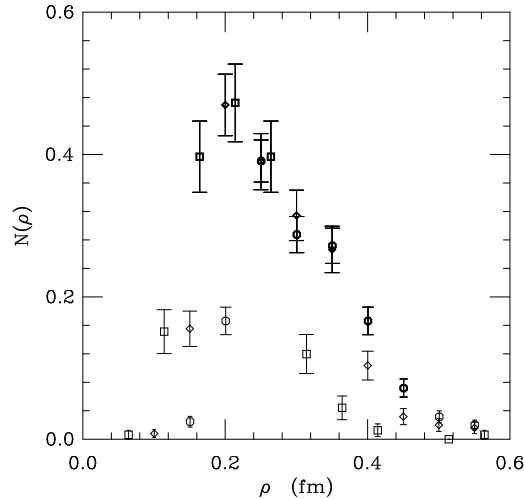


Figure 2. The density distribution of instantons. Data at $a = 0.116(2)$ fm are given by squares, $a = 0.144(1)$ fm diamonds, and $a = 0.188(3)$ fm octagons. The bold data points are ones for which the instanton radius is large compared to the lattice spacing and small compared to the simulation volume.

as they were found on the smoothed configurations, and then to compare the physical properties of these artificial configurations with the real smoothed ones. As an illustration in Fig. 3 we show the heavy quark potential measured with timelike Wilson loops. The comparison shows that instantons are not very likely to be responsible for confinement. Since we have no information about the relative orientation of instantons in group space, in the artificial configurations we just put them aligned. Work is in progress to study the case of randomly oriented instantons.

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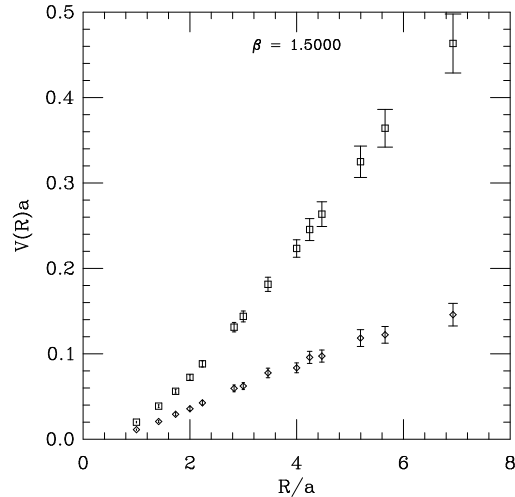


Figure 3. The heavy quark potential measured on the artificially produced instanton configurations (diamonds) and the potential measured on the corresponding 9 times smoothed “real” configurations (squares), at $\beta = 1.5$.

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